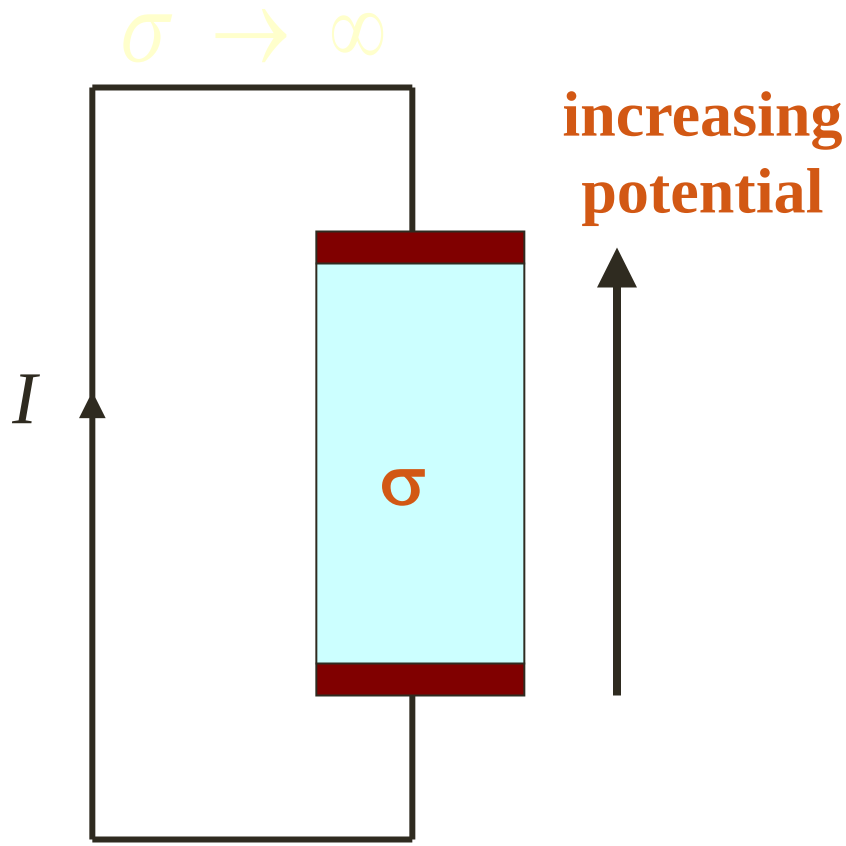
Electromotive Force & Continuity Equation

Lecture 6

Electromotive Force

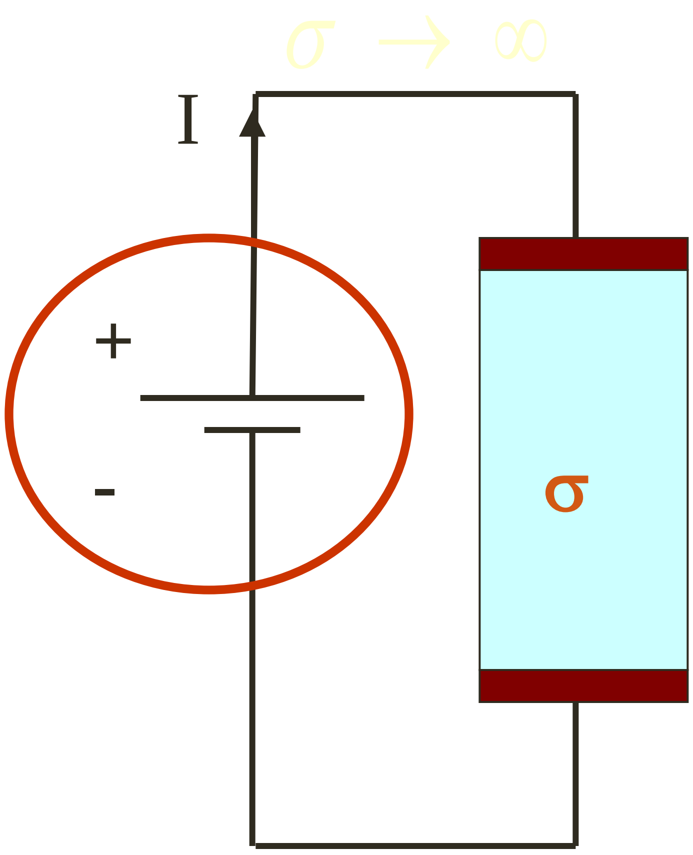
- Steady current flow requires a closed circuit.
- Electrostatic fields produced by stationary charges are conservative. Thus, they cannot by themselves maintain a steady current flow.

Electromotive Force (Cont'd)



- The current *I* must be zero since the electrons cannot gain back the energy they lose in traveling through the resistor.

Electromotive Force (Cont'd)



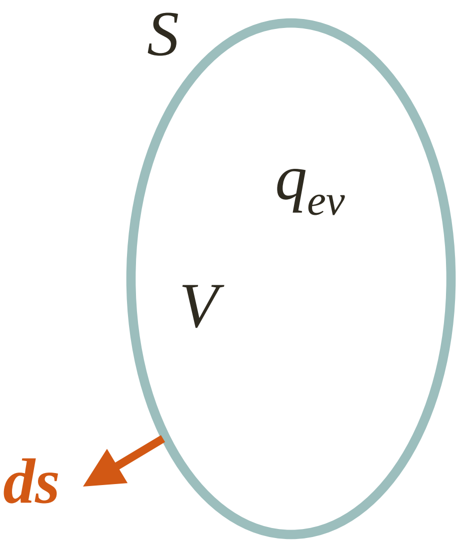
- To maintain a steady current, there must be an element in the circuit wherein the potential rises along the direction of the current.

Conservation of Charge

- Electric charges can neither be created nor destroyed.
- Since current is the flow of charge and charge is conserved, there must be a relationship between the current flow out of a region and the rate of change of the charge contained within the region.

Conservation of Charge (Cont'd)

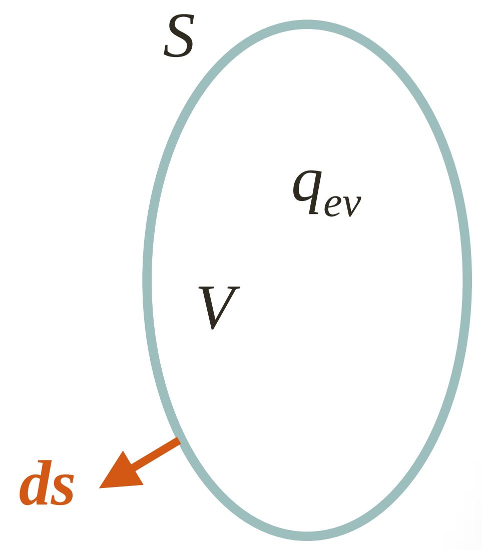
- Consider a volume V bounded by a closed surface S in a homogeneous medium of permittivity ε and conductivity σ containing charge density q_{ev}.



Conservation of Charge (Cont'd)

- The net current leaving V through S must be equal to the time rate of decrease of the total charge within V, i.e.,

$$I = -\frac{dQ_{enc}}{dt}$$



Conservation of Charge (Cont'd)

- The net current leaving the region is given by
- The total charge enclosed within the region is given by

$$I = \oint_S \underline{J} \cdot d\,\underline{s}$$

$$Q = \int_V q_{ev}\, dv$$

Conservation of Charge (Cont'd)

- Hence, we have

$$\oint_S \underline{J} \cdot d\underline{s} = -\frac{d}{dt}\int_V q_{ev}\, dv$$

net outflow of current

net rate of decrease of total charge

Continuity Equation

- Using the *divergence theorem*, we have

$$\oint_S \underline{J} \cdot d\,\underline{s} = \int_V \nabla \cdot \underline{J}\, dv$$

- We also have

$$\frac{d}{dt}\int_V q_{ev}\, dv = \int_V \frac{\partial q_{ev}}{\partial t}\, dv$$

Becomes a partial derivative when moved inside of the integral because q_{ev} is a function of position as well as time.

Continuity Equation (Cont'd)

- Thus,
- Since the above relation must be true for *any and all regions*, we have

$$\int_V \nabla \cdot \underline{J}\, dv + \int_V \frac{\partial \rho}{\partial t}\, dv = 0$$

$$\boxed{\nabla \cdot \underline{J} + \frac{\partial \rho}{\partial t} = 0}$$

Continuity Equation

Continuity Equation (Cont'd)

- For steady currents,

$$\frac{\partial \rho}{\partial t} = 0$$

- Thus,

$$\boxed{\nabla \cdot \underline{J} = 0}$$

J is a *solenoidal* vector field.

Continuity Equation in Terms of Electric Field

- Ohm's law in a conducting medium states

$$\underline{J} = \sigma \underline{E}$$

- For a homogeneous medium

- But from Gauss's law,

$$\nabla \cdot \underline{J} = \sigma \nabla \cdot \underline{E} = 0 \quad \Rightarrow \quad \nabla \cdot \underline{E} = 0$$

- Therefore, the volume charge density, ρ, must be zero in a homogeneous conducting medium

$$\nabla \cdot \underline{E} = \frac{q_{ev}}{\varepsilon}$$